

First Prize Winner Ms. Madhumitha's Solution

Given :

ΔABC is inscribed in a circle. AD is altitude from A .

Construction :

CG is altitude from C and let O be orthocentre.

Join MD .

Given : M, L are midpoints of AB & BC respect.

We know that nine point circle passes through foot of perpendicular, midpoint of side

$\Rightarrow M, G, D, L$ lies in Nine points circle

To prove that F also lies on the same circle.

From Novelties of Geometry page 15 (proof for novelty 3) We know that $OD=DE$

$$\text{As } DF = \frac{1}{2} AE$$

$$\Rightarrow DF = DE + AF \text{ -----(1)}$$

$$DF = DO + OF$$

$$= DE + OF \text{ -----(2)}$$

from (1) & (2) it is clear that $OF = AF$

$\Rightarrow F$ lies on 9 point circle

[3 midpoints sides 3 foot of perpendicular, 3 midpoints from vertex to orthocentre]

$\Rightarrow MFLD$ is concyclic

$\Rightarrow \angle MFL = \angle MDB$ (In cyclic quadrilateral exterior angle is equal to opposite interior angle)

Consider right ΔADB .

As $AM = MB$ (M is midpoint of AB)

M is circumcenter of $\Delta ADB \Rightarrow AM = MB = MD$.

$\Rightarrow \Delta MBD$ is isosceles

$\Rightarrow \angle MDB = \angle MBD$.

Thus $\angle MFL = \angle B$.

